Optical Engineering

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Course outline

1. Imaging optics
   2.1 Light propagation across interfaces
   2.2 Photography and optical lenses
   2.3 Aberrations
   2.4 Plane parallel plates and reflective prisms
   2.5 Depth of focus
   2.6 Magnifying glass and basic microscope
   2.7 Modern microscopes
   2.8 Beam expansion
   2.9 Telescopes
   2.10 Optic design
2. Optical sensors
3. Optics in data storage
4. Introduction to displays
5. Fourier optics
6. Diffractive optics and holograms
7. Integrated optics
8. Computerized imaging

Exercise: Selection of telescope

• Collect the advantages and disadvantages of the two telescopes!
• Decide as a group on one telescope!

Comparison of telescopes

BRASKO 60700 Telescope
- Refractor
- Terrestrial observation

Bresser Telescope Pluto 114/500
- Reflector
- No spherical/chromatic aberrations
- Mirror diameter larger, thus more light
- More compact
- More expensive
- Prettier
- Aberrations due to diffraction on second mirror
### Important factors for telescope purchase

- **Two main tasks of telescope:**
  - Magnify small objects
  - Brighten faint objects

- **Useful magnification limited by diffraction**
  - Max. Magnification = Diameter of objective in mm
  - Image may be magnified more, but image information is limited
  - Analog: Not more detail on newspaper image observable with magnifying glass

- **Aberrations and air turbulence ("Seeing") limit magnification further**
  - Max. resolution of terrestrial telescope \( \approx 1" \)
  - Hubble-Telescope \( \approx 0.05" \) at visible wavelengths

- **Larger aperture for brighter images!**

- **Quality of mount is also critical**
  - Should not move more than 1 sec after touching

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#### 2.10 Optic design
- Analytical solution (optical quadrupole)
- Matrix formulation
- Sequential raytracing
- Non-sequential raytracing
- FDTD

### Geometrical optics / Ray optics

- Rays describe propagation of light in (most) optical systems sufficiently well, if dimensions \( X \) of objects and parts sufficiently larger than:
  - Wavelength \( \lambda \ll X \)
  - Coherence length \( \xi \ll X \)

- Wave properties of light neglected
  - No interference, diffraction, near field effects ...

- Geometrical optics may be derived from wave optics for wavelength approaching zero

- Direction of ray propagation is direction of wavevectors

### Paraxial approximation / Gaussian optics

- For propagation at small angles relative to optical axis simplified description of optical parts is possible and analytical solutions exist.

\[
n_1 \cdot \sin \alpha = n_2 \cdot \sin \beta
\]

\[
n_1 \cdot \alpha = n_2 \cdot \beta
\]
### 4.9 Spherical surface in paraxial approximation

- Approximated

\[
s' \approx \frac{n'}{n-n} \frac{n}{r} \frac{n}{s}
\]

- For rays parallel to optical axis

\[
s' \approx \frac{n'}{n'-n} r
\]

### 4.10 Thick lens with spherical surfaces

- Calculation of optical power of thick lens
- Optical power

\[
D = -\frac{n}{f}
\]

- \([D]=m^{-1}=\text{Diopter=dpt}\)

### 4.11 Thick lens: Focal point

- Focal length is distance from principle point to focal point

\[
s_1' = \frac{n_1}{n_1 - 1} r_1
\]

\[
s_2 = s_1' - d = \frac{n_L}{n_L - 1} r_1 - d = \frac{n_L r_1 - n_L d + d}{n_L - 1}
\]

\[
s_2' = 1 \frac{1 - n_L}{r_2} + \frac{n_L}{n_L - 1} r_2 = \frac{1 - n_L}{n_L - 1} \frac{1}{r_2} \frac{n_L (n_L - 1)}{n_L r_1 - n_L d + d}
\]

\[
= \frac{r_2 (n_L r_1 - n_L d + d)}{(n_L - 1) [n_L (r_2 - r_1) + d (n_L - 1)]}
\]

### 4.12 Front principle plane

- Front principle plane: Locus of intersection of diverging/converging bundle of rays for parallel outgoing rays
- Front principle point H: Intersection of front principle plane with optical axis

**Positive (converging) system**

**Negative (diverging) system**

Source: Schröder/Treiber, *Technische Optik*
4.13 Rear principle plane

- Rear principle plane: Locus of intersection of parallel incoming rays with diverging/converging outgoing rays
  - Rear principle point $H'$: Intersection of rear principle plane with optical axis

4.14 Thick lens: Focal length and optical power

\[
f = s_1 \cdot s_2' = \frac{n_r r_2}{(n_r - 1)(n_r (r_2 - r_1) + d(n_r - 1))}
\]

\[
D = -\frac{(n_r - 1)(n_r (r_2 - r_1) + d(n_r - 1))}{n_r r_2}
\]

4.15 Possible lens shapes

4.16 Special case: Plano-convex and Plano-concave lenses

- For plane surface $r_2 \rightarrow \infty$

- This results in

\[
f = \frac{r_1}{n_r - 1}
\]
  
  - Focal length independent of $d$!
Special case: Spherical lens

- For spherical surface \( r_1 = r = -r_2 \) \( d = 2r \)
- This results in

\[
 f = \frac{n_L}{n_L - 1} \frac{r}{2}
\]

Special case: Thin lens

- For negligible \( d \)
- This results in

\[
 f = \frac{r r_2}{(n_L - 1)(r_2 - r_1)} \\
 \frac{1}{f} = (n_L - 1) \left( \frac{1}{r_1} - \frac{1}{r_2} \right) \\
 HH' = d \left( \frac{n_L - 1}{n_1} \right)
\]

For thin lenses distance between principle planes may be neglected.

Notation for imaging surfaces

Equivalent lens

- System with several stages (e.g. objective with several lenses) may be described by optical quadrupole (“Equivalent lens”)
  - Normally distance of individual systems measured between \( H_1' \) und \( H_2 \)
  - In case of microscopes tube length between focal points used

Source: Schröder/Treiber, Technische Optik
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       • Analytical solution (optical quadrupole)
       • Matrix formulation
       • Sequential raytracing
       • Non-sequential raytracing
       • FDTD

Matrix optics / ABCD-matrices

- Propagation of ray at point $A$ completely described by distance $x$ to optical axis and angle $\theta$ relative to optical axis

\[ \vec{s}_1 = \begin{pmatrix} x_i \\ \theta_i \end{pmatrix} \]

\[ \vec{s}_2 = M_{12} \cdot \vec{s}_1 \]

Optical system with several components

- Effect of total system

\[ \vec{s}_{\text{Output}} = \prod_i M_j \cdot \vec{s}_{\text{Input}} \]

1. Matrix of translation

\[ x_2 = 1 \cdot x_1 + L \cdot \theta_1 \]

\[ \theta_2 = 0 \cdot x_1 + 1 \cdot \theta_1 \]

\[ \vec{s}_2 = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \cdot \vec{s}_1 = M_{\text{Translation}} \cdot \vec{s}_1 \]
2. Refraction on plane interface

\[ x_2 = 1 \cdot x_1 + 0 \cdot \theta_1 \]
\[ \theta_2 = 0 \cdot x_1 + \frac{n_1}{n_2} \cdot \theta_1 \]

\[ \bar{s}_2 = \begin{pmatrix} 1 & 0 \\ 0 & \frac{n_1}{n_2} \end{pmatrix} \cdot \bar{s}_1 = M_{\text{Plane interface}} \cdot \bar{s}_1 \]

Exercise: Thin lens

- All important simple optical elements may be described using the tree matrices (Translation, plane interface and spherical interface)
  - Thin lens (convex, concave)
  - Thick lens (convex, concave)
  - Mirror (plane, focusing)

- Derive the total matrix for a thin lens!
- Derive the dependence of the focal length \( f \) and the interface curvatures (Lens maker's equation for thin lens)!
- Derive the matrix for imaging with a convex lens!
- Derive the imaging equation from the resulting matrix!
  - Consider that \( x_2 \) has to be angle-independent

3. Refraction on spherical interface

- Snellius at interface
- Angle depends on radius
  - Convention for curvature and propagation \( \rho > 0 \)

\[ \bar{s}_2 = \begin{pmatrix} 1 & 0 \\ \frac{n_1 - n_2}{n_2 \rho} & \frac{n_1}{n_2} \end{pmatrix} \cdot \bar{s}_1 = M_{\text{Spherical interface}} \cdot \bar{s}_1 \]

Thick lens

- Combination of two spherical interfaces and translation
  - Focal length calculated from principal plane

\[ \bar{s}_2 = M_{\text{SI}} (\rho_2, n_2 \rightarrow n_1) \cdot M_{\text{Tr}} (L) \cdot M_{\text{SI}} (\rho_1, n_1 \rightarrow n_2) \cdot \bar{s}_1 \]
Matrix properties in Gaussian optics

\[ M_{12} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \]

- Because of Snellius law:
  \[ \det M_{12} = AD - BC = \frac{n_1}{n_2} \]

  - Only 3 of the 4 matrix elements may be chosen independently

Optical elements and their system matrices

**Focusing:**

\[ A = 0 \Rightarrow x_2 = B \theta_1 \]

\[ \begin{pmatrix} 0 & B \\ C & D \end{pmatrix} \]

**Optical imaging:**

\[ B = 0 \Rightarrow x_2 = A x_1 \]

\[ \begin{pmatrix} A & 0 \\ C & D \end{pmatrix} \]

**Deflection of parallel rays:**

\[ \theta_2 = D \theta_1 \]

\[ \begin{pmatrix} A & B \\ 0 & D \end{pmatrix} \]

**Generation of parallel rays:**

\[ \theta_2 = C x_1 \]

\[ \begin{pmatrix} A & B \\ C & 0 \end{pmatrix} \]

Matrix optics for Gaussian beams and polarization

- Matrix optics not limited to Gaussian optics, but may be used for Gaussian beams as well.
  - ABCD-Matrices identical
  - Instead of ray vector \( s \) beam parameter \( q \) used:
    \[
    q(z) = \frac{A q_0 + B}{C q_0 + D} \quad 1 = \frac{1}{q} = \frac{i \lambda}{R \pi \omega^2}
    \]

  - Polarization may be included as well in matrix algorithm
    - Jones-vector describes state of polarization
    - Jones-matrices describe optical elements

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4.33 Sequential ray-tracing

- Rays propagates through optical elements in sequential order
- Used for design of imaging optical systems
  - Microscope, telescope, camera ...


4.34 Non-sequential ray-tracing

- Rays split on surface and may pass optical elements several times
- By propagation of Gaussian beams wave phenomena may be included
- Used for design including stray light calculation
  - Stray light calculation
  - Back lights, luminaires ...


4.35 FDTD: Finite Difference Time Domain

- Solution of Maxwell-equations using space and time grid
- Used for micro- and nanosystems
  - Integrated optics, photonic crystals, plasmonics ...


4.36 Common optical design software

<table>
<thead>
<tr>
<th>A selection of programs to consider</th>
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<tbody>
<tr>
<td><strong>Program</strong></td>
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<tr>
<td><strong>Sequential ray-tracing programs</strong></td>
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<td>Opticis®</td>
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<td>OSLO®</td>
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<td>ZEMAX®</td>
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<td><strong>Non-sequential ray-tracing programs</strong></td>
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<td>OptiFDTD®</td>
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- At the university additionally COMSOL Multiphysics

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1. Imaging optics
2. Optical sensors
   2.1 Spectroscopy
   2.2 Material characterization
   2.3 Distance measurement
   2.4 Angle measurement
   2.5 Optical mouse
3. Optics in data storage
4. Introduction to displays
5. Fourier optics
6. Diffractive optics and holograms
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Spectroscopy

Spectroscopy: Frequency or wavelength analysis of light

Applications of spectroscopy
- Material characterization
- Sensors
- Development of light sources

Example: Organic light emitting diode

Optical spectrum characteristic for chemical elements
- Known since appr. 1859 (Kirchhoff, Bunsen), e.g.:
  - Na
  - He
  - e.g.: Fraunhofer-lines in solar spectrum (1802, 1814) (Absorption spectra)

In general:
- Glowing solid or liquid objects have continuous spectra (e.g. thermal radiation, black body, Planck's law)
- Glowing gases or vapors have discontinuous spectra (Transition between discrete energy levels in atoms/molecules)


Spectrometer using absorptive filters

• Spectrally broad CCD-chip is illuminated through sequence of filters

Source: http://www.technoteam.de/
4.41 Spectrometers using dispersive elements

- **Goal:** Spatial separation of light of different wavelengths
e.g., on a screen or a CCD-chip

- **Use dispersive element**
  Material or structure whose effect on light is wavelength-dependent(e.g. prisms, grating)

- **Common types of spectrometers sorted by dispersive element**
  - Prism spectrometer uses dispersion due to refraction
  - Grating spectrometer uses dispersion due to diffraction

- **Common types of spectrometers sorted by functionality**
  - Monochromator selects small wavelength interval
  - Spectrometer for observation of large wavelength interval
  - Spectrograph Spectrometer + CCD-camera / film etc.

4.42 Material dispersion

- **Refractive index depends on type of glass and on wavelength dependent**

![Graph showing refractive index vs wavelength for different types of glass](http://en.wikipedia.org)

4.43 Prism spectrometer – Refraction in prism

Deflection angle: $\delta = \theta - \alpha + \arcsin(n \sin \alpha \sqrt{n^2 - \sin^2 \theta - \cos \alpha \sin \theta})$

Dispersion: $n = n(\lambda) \implies \delta = \delta(\lambda)$

Minimum deflection angle for symmetric pass:

$\delta_{\text{min}} = 2 \left(\arcsin(n \sin \frac{\theta}{2}\right) - \alpha$

Angular dispersion $\frac{d\delta}{d\lambda} = \frac{2 \sin \frac{\theta}{2}}{\sqrt{1 - n^2 \sin^2 \frac{\theta}{2}}} \cdot \frac{d\lambda}{\lambda}$

Source: *Wikipedia* and *Optik Licht und Laser* von Dieter Meschede

4.44 Prism spectrometer

**Advantage:** Unambiguous correlation between wavelength and position in image plane

**Disadvantage:** Small dispersion and thus small resolution

$\frac{\Delta \lambda}{\Delta} = b \cdot \frac{d\lambda}{\lambda}$

Basis of prism

Source: *Bauelemente der Optik* von Naumann und G. Schröder
Interference – special case

Interference: Superposition of waves with fixed phase relation. Interference is due to wave character of light and cannot be understood using ray optics.

Example: Two monochromatic waves of identical polarization and identical amplitude are superposed at position $\vec{r}$

$$\vec{E}_1 = A \exp \left( j(\omega t - k_1 r - \phi_1) \right)$$

$$\vec{E}_2 = A \exp \left( j(\omega t - k_2 r - \phi_2) \right)$$

Principle of superposition for fields: $\vec{E} = \vec{E}_1 + \vec{E}_2$

Irradiance (intensity) at point $\vec{r}$ is:

$$I(\vec{r}) = 2A_0 \left[ 1 + \cos \left( (\vec{k}_1 - \vec{k}_2) \cdot \vec{r} - (\phi_1 - \phi_2) \right) \right]$$

Spatially modulated interference pattern with minima and maxima

Interference term results as

$$I(\vec{r}) = \frac{\varepsilon_0 c}{2} \left[ |\vec{E}_1|^2 + |\vec{E}_2|^2 + \Re \{\vec{E}_1 \cdot \vec{E}_2^*\} \right]$$

Interference depends on polarization $\sim \vec{e}_1 \cdot \vec{e}_2$

Constructive / destructive interference

$$\Re \{\vec{E}_1 \cdot \vec{E}_2^*\} = A_1 \cdot A_2 \cdot \vec{e}_1 \cdot \vec{e}_2 \cdot \cos \left[ \left( (\vec{k}_1 - \vec{k}_2) \cdot \vec{r} - (\phi_1 - \phi_2) \right) \right]$$

Phase difference $\delta = (\vec{k}_2 - \vec{k}_1) \cdot \vec{r} - (\phi_1 - \phi_2)$

Path difference $\Delta = \delta_{\lambda} + \Delta'$

Difference of opt. paths of two partial waves $\Delta = m \cdot \lambda$

Constructive interference / Maximum of intensity for:

$$\delta = m \cdot 2\pi \quad m = 0, \pm 1, \pm 2, \pm 3, ..$$

Destructive interference / Minimum of intensity for:

$$\delta = n \cdot 2\pi \quad n = \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{5}{2}, ..$$

Grating principle

- Interference of spherical waves exiting different slits
- Each slit also causes diffraction

Maxima, wenn $\Delta s = d \sin \theta = m \lambda$

$m = 0, 1, 2$

Formula for N slits:

$$I(\theta) = \frac{1}{N^2} \cdot \sin^2 \left[ N \pi (d/\lambda) \sin \theta \right] \cdot \frac{\sin^2 \pi b/\lambda \sin \theta}{\sin^2 \left[ \pi (d/\lambda) \sin \theta \right]}$$

Source: Optics lecture by PD Dr. Seifert, Universität Halle (http://www.physik.uni-halle.de/)
Grating - intensity with angle

- Main maxima for constructive interference of all wave parts

Example:
N = 8,
d = 2b

Grating equation

- Maximum intensity at:

\[ \sin \theta_m - \sin \theta_0 = \pm \frac{m \lambda}{d} \]

Grating ambiguity

- Gratings are ambiguous!

Exercise: Grating period

- Determine the grating period of the given grating!
Grating – width of maximum

- For more illuminated slits maximum is narrower

\[ \Delta \theta = \frac{\lambda}{N d \cos \theta_m} = \frac{\lambda}{N d} \]

Source: Optics lecture by PD Dr. Seifert, Universität Halle (http://www.physik.uni-halle.de/)

Grating resolution

- Two wavelengths are resolved if they are at least separated by width of main maximum

\[ \sin \theta_m = \frac{m \cdot \lambda}{d} \]

\[ m \cdot \lambda = \frac{d}{\Delta \lambda} = \sin \theta_m - \sin \theta_i = 2 \cos \frac{\theta_i + \theta_d}{2} \sin \frac{\theta_i - \theta_d}{2} = \Delta \theta \cdot \cos \left( \frac{\Delta \theta}{2} \right) \]

\[ \Delta \theta \geq \Delta \theta = \frac{\lambda}{N d \cos \theta_m} \Rightarrow \frac{m \cdot \lambda}{N d} \Delta \lambda = \frac{\lambda}{N d \cos \theta_m} = \frac{\lambda}{N d} \]

Resolution in order \( m \) for grating for \( N \) illuminated slits

Source: Optics lecture by PD Dr. Seifert, Universität Halle (http://www.physik.uni-halle.de/)

Types of gratings

- Amplitude grating
  Slits as origin of fundamental waves.

- Phase grating
  Modulation of optical path! Modulation of phase

- Reflection grating
  Often blazed

Blazed gratings

- In Littrow configuration groove have such profiles that incident and diffracted rays are in auto collimation (i.e., \( \alpha = \beta \)).
  - Efficiency of grating increased
- In this case at the "blaze" wavelength \( \lambda_B \) we obtain:

\[ \beta = -\alpha \]

\[ 2 \sin \alpha = \frac{m \lambda_B}{d} \]

Source: http://www.jobinyvon.com
Czerny-Turner monochromator

- Light leaving the exit slit (G) contains entire image of entrance slit of selected color plus parts of entrance slit images of nearby colors.
- Rotation of dispersing element causes band of colors to move relative to exit slit.

\[ \frac{d\theta}{d\lambda} = \frac{m}{d \cos \theta_m} \]

Source: http://en.wikipedia.org

Échelle spectrographs

- Prism for dispersion in one direction
- Grating for dispersion in orthogonal direction

Source: http://www.andor.com

Comparison grating / prism spectrometer

- Reflection grating
  - 1200 rules/mm
  - Width 60mm
- Prism
  - Glass SF11

Resolution at 550nm

\[ n = 1,792 \]
\[ b = 60 \text{mm}, \alpha = 60^\circ \]
\[ \frac{dn}{d\lambda} = 2 \times 10^{-4} \text{nm}^{-1} \]
\[ \frac{d\alpha}{d\lambda} = 16 \times 10^{-4} \text{nm}^{-1} \]
\[ \frac{\lambda}{\Delta \lambda} = 72000 \]
\[ \frac{\lambda}{\Delta \lambda} = 12000 \]

Exercise: Spectroscope

- Build the spectroscope!
- Observe and sketch the spectrum of an incandescent source and a fluorescent lamp!
  - What influence does the entrance slit have?
Compilation of questions

- When may Gaussian optics be used?
- How are the principle planes of a thick lens defined?
- What is a thin lens?
- What are ABCD-matrices and what are they used for?
- What is the difference between sequential and non-sequential raytracing?
- How does a spectrometer using absorptive filters work?
- Sketch a prism spectrometer!
- What is constructive interference?
- Sketch the outgoing intensity as a function of angle for a grating!
- Why is the wavelength determination with a grating ambiguous?
- What limits the resolution of a grating monochromator?
- Sketch a grating spectrograph!